Nonlinear Correction Effects on Transverse Dust Lattice Waves in Dusty Plasmas

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The nonlinear correction effects on the transverse dust lattice mode wave are investigated in dusty plasmas. It is shown that the nonlinear correction effects on the dispersion relation increase with decreasing a/λ_{D}, where a is the spacing of dust grains and a/λ_{D} is the Debye length. It is also found that the nonlinear correction effects enhance the frequency of the transverse dust lattice wave.

Key words: Transverse Dust Lattice Waves; Dusty Plasmas.

Recently, there has been a considerable interest in the dynamics of strongly coupled plasmas containing charged dust grains including nonideal collective effects, nonlinear effects, and electrostatic interactions. It was known that the dust-plasma interactions occur not only in space plasmas but also in the laboratory plasmas. Various physical processes in dusty plasmas have been investigated in order to obtain information on relevant plasma parameters [1 – 7]. Recent investigations [6, 8, 9] have predicted the existence of transverse dust lattice mode waves in strongly coupled dusty plasmas. The particle interaction potentials in dusty plasmas have been represented as the Debye-Hückel potential [1, 10] obtained by linearization of the Poisson equation with the Boltzmann distribution function. However, in the case of highly charged dust grains the description of the potential would not be described by the standard Debye-Hückel potential due to strong nonlinear electrostatic interactions. Hence, the dust lattice wave in crystallized dust strings would be affected by these nonlinear correction effects. To the best of our knowledge, the nonlinear effects on the dispersion relation of the dust lattice mode wave in dusty plasmas have not yet been investigated. Thus, in this paper we investigate the nonlinear correction effects on the transverse dust lattice mode wave in strongly coupled dusty plasmas using a nonlinear screened potential. The dispersion relation of the transverse dust lattice wave, including the nonlinear correction effects, is obtained.

Very recently, the modified screening interaction potential in dusty plasmas has been given by the nonlinear screened potential [11, 12] using the nonlinear corrections in the expanded Boltzmann distribution. If we assume that the dusty strings are composed of the same dust grains, the total interaction potential U_{ij} between two charged dust grains using this nonlinear screened potential model in strongly coupled dusty plasmas with charges $Q_{\mathrm{D}}(=-Z_{\mathrm{D}}e)$ at the positions $\mathbf{r}_i$ and $\mathbf{r}_j$ would be represented by

$$\begin{aligned} U_{ij}(\mathbf{r}_i,\mathbf{r}_j) &= \frac{Q_{\mathrm{D}}^2}{|\mathbf{r}_i-\mathbf{r}_j|}\exp\left(-|\mathbf{r}_i-\mathbf{r}_j|/\lambda_{\mathrm{D}}\right) \\ &-\frac{c}{4}\frac{Q_{\mathrm{D}}^2}{|\mathbf{r}_i-\mathbf{r}_j|}\exp\left(-|\mathbf{r}_i-\mathbf{r}_j|/\lambda_{\mathrm{D}}\right)\mathrm{Ei}\left(-3|\mathbf{r}_i-\mathbf{r}_j|/\lambda_{\mathrm{D}}\right) \\ &+\frac{c}{4}\frac{Q_{\mathrm{D}}^2}{|\mathbf{r}_i-\mathbf{r}_j|}\exp\left(-|\mathbf{r}_i-\mathbf{r}_j|/\lambda_{\mathrm{D}}\right)\mathrm{Ei}\left(-|\mathbf{r}_i-\mathbf{r}_j|/\lambda_{\mathrm{D}}\right), \end{aligned} \quad (1)$$

where $c \equiv Q_{\mathrm{D}}\mathrm{e}/\lambda_{\mathrm{D}}T$, T is the plasma temperature, and $\mathrm{Ei}(-r/\lambda_{\mathrm{D}}) = -\int_{r/\lambda_{\mathrm{D}}}^{\infty}\mathrm{d}t\,\mathrm{e}^{-t}/t$ is the exponential integral [13], λ_{D} $(\equiv \lambda_{\mathrm{De}}\lambda_{\mathrm{Di}}/\sqrt{\lambda_{\mathrm{De}}^2+\lambda_{\mathrm{Di}}^2})$ is the dusty plasma Debye length [6], λ_{De} and λ_{Di} are the electron and ion Debye radii, respectively. If we neglect the nonlinear correlation terms, the interaction potential U_{ij} becomes the Debye-Hückel potential $U_{ij}^{\mathrm{DH}} = (Q_{\mathrm{D}}^2/|\mathbf{r}_i-\mathbf{r}_j|)\exp(-|\mathbf{r}_i-\mathbf{r}_j|/\lambda_{\mathrm{D}})$. The transverse plasma oscillations in dust strings in strongly

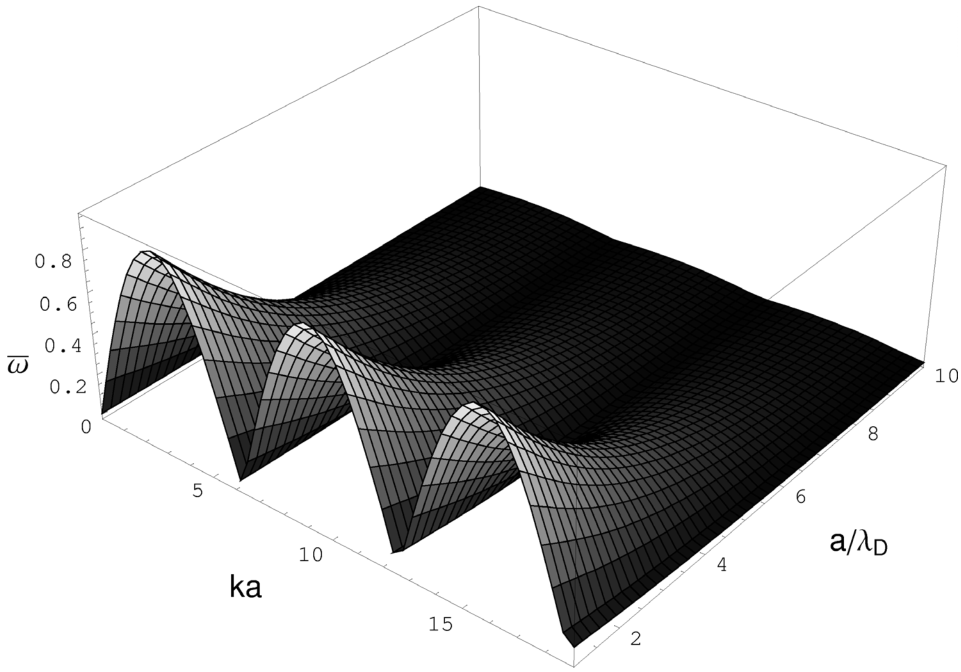

Fig. 1. Three-dimensional plot of the scaled frequency $(\bar{\omega})$ as a function of the scaled wave number (ka) and the scaled reciprocal Debye length (a/λ_{D}) when $\xi = 1$.

coupled dusty plasmas are represented by the equation of motion of the perturbed dust grains:

$$\frac{\mathrm{d}^2 \delta z_j(t)}{\mathrm{d}t^2} = -\sum_i M_{\mathrm{D}}^{-1} \frac{\partial U_{ij}(\mathbf{r}_i, \mathbf{r}_j)}{\partial z_j}, \tag{2}$$

where $z_j[= z_{j0} + \delta z_j(t)]$ is the position of the jth dust grain, z_{j0} is the equilibrium position of the jth grain, $\delta z_j(t)$ is the vertical displacement of the jth grain, and M_{D} is the mass of the dust grain. Here, we assume that the dust string is composed of an equally spaced chain of equal dust grains. After some algebra taking into account the nearest-neighbor interactions ($|i - j| = 1$), i. e., $i = j - 1$ and $j + 1$, we have

$$\begin{aligned}\left(\sum_{i=j-1,j+1} \frac{\partial U_{ij}}{\partial z_j}\right)_{|z_i - z_j| = a} &= \left(\delta z_{j+1} + \delta z_{j-1} - 2\delta z_j\right) \\ &\cdot \left\{ \left(-\frac{Q_{\mathrm{D}}^2}{a^3} - \frac{Q_{\mathrm{D}}^2}{a^2 \lambda_{\mathrm{D}}}\right) \mathrm{e}^{-a/\lambda_{\mathrm{D}}} \right. \\ &- \frac{c}{4} \left[\left(-\frac{Q_{\mathrm{D}}^2}{a^3} + \frac{Q_{\mathrm{D}}^2}{a^2 \lambda_{\mathrm{D}}}\right) \mathrm{e}^{a/\lambda_{\mathrm{D}}} \mathrm{Ei}(-3a/\lambda_{\mathrm{D}}) \right. \\ &\left.\left. + \left(\frac{Q_{\mathrm{D}}^2}{a^3} + \frac{Q_{\mathrm{D}}^2}{a^2 \lambda_{\mathrm{D}}}\right) \mathrm{e}^{-a/\lambda_{\mathrm{D}}} \mathrm{Ei}(-a/\lambda_{\mathrm{D}}) \right] \right\},\end{aligned} \tag{3}$$

where $a(= |z_{j+1} - z_j| = |z_j - z_{j-1}|)$ is the separation between two consecutive dust grains. If we assume the vertical perturbation as $\delta z_j(t) = z_{j0} \exp[-\mathrm{i}(\omega t - jka)]$, where ω and k are the frequency and wave number, the dispersion relation is then found to be

$$\begin{aligned}\omega^2(k, a, \lambda_{\mathrm{D}}, \xi) &= \frac{4Q_{\mathrm{D}}^2}{M_{\mathrm{D}} a^3} \sin^2(ka/2) \left\{ \left(1 + \frac{a}{\lambda_{\mathrm{D}}}\right) \mathrm{e}^{-a/\lambda_{\mathrm{D}}} \right. \\ &- \xi \left(\frac{a}{\lambda_{\mathrm{D}}}\right) \left[\left(-1 + \frac{a}{\lambda_{\mathrm{D}}}\right) \mathrm{e}^{a/\lambda_{\mathrm{D}}} \mathrm{Ei}(-3a/\lambda_{\mathrm{D}}) \right. \\ &\left.\left. + \left(1 + \frac{a}{\lambda_{\mathrm{D}}}\right) \mathrm{e}^{-a/\lambda_{\mathrm{D}}} \mathrm{Ei}(-a/\lambda_{\mathrm{D}}) \right] \right\},\end{aligned} \tag{4}$$

where $\xi \equiv Z_{\mathrm{D}} \mathrm{e}^2/4aT$. This dispersion relation defines the transverse dust lattice mode wave including the nonlinear correction effects and is reliable since it would be sufficient. However, if we neglect the nonlinear correction terms in (1), i. e., using the Debye-Hückel potential, the dispersion relation [6, 14] is then obtained as

$$\omega_0^2 = (4Q_{\mathrm{D}}^2/M_{\mathrm{D}} a^3) \sin^2(ka/2)(1 + a/\lambda_{\mathrm{D}}) \mathrm{e}^{-a/\lambda_{\mathrm{D}}}. \tag{5}$$

Figure 1 shows the three-dimensional plot of the scaled wave frequency $\bar{\omega}$ $(\equiv \omega/2Q_{\mathrm{D}} M_{\mathrm{D}}^{-1/2} a^{-3/2})$

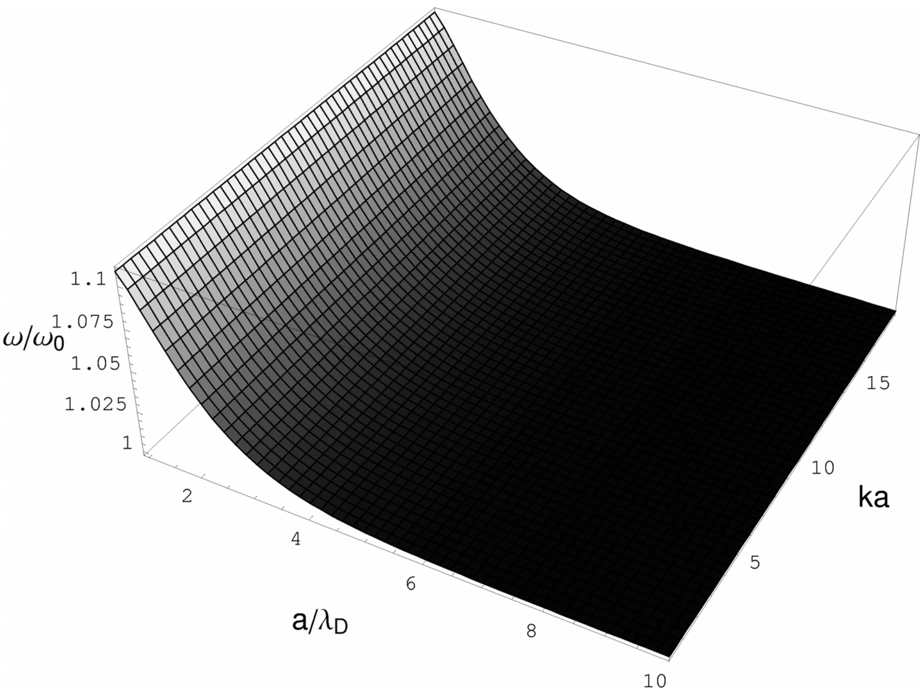

Fig. 2. Surface plot of ω/ω_0 as a function of the scaled reciprocal Debye length (a/λ_D) and the scaled wave number (ka) when $\xi = 1$.

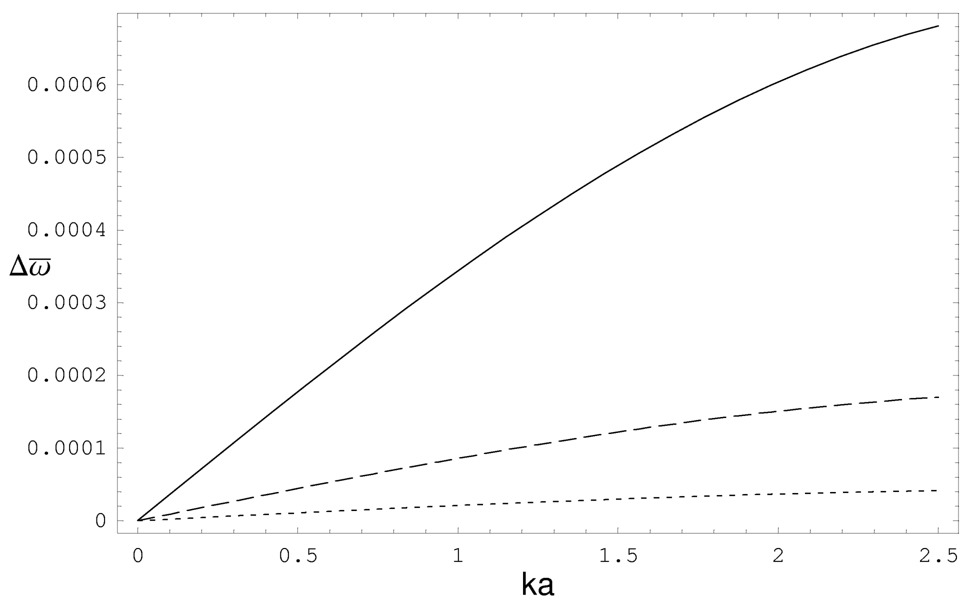

Fig. 3. Plot of the frequency difference $(\Delta\bar{\omega})$ for small wave numbers when $\xi = 1$. The solid, dashed and dotted curves represent the cases $a/\lambda_D = 5$, 6 and 7, respectively.

as a function of the scaled wave number (ka) and the scaled reciprocal Debye length (a/λ_D). From this figure we realized that the wave frequency decreases exponentially with a/λ_D, since the function $(-1 + a/\lambda_D)e^{a/\lambda_D}\mathrm{Ei}(-3a/\lambda_D) + (1 + a/\lambda_D)e^{-a/\lambda_D}\mathrm{Ei}(-a/\lambda_D)$ increases with a/λ_D. It is also found that the wave frequency is oscillating with ka due to the factor $|\sin(ka/2)|$ in (4). Figure 2 represents the three-dimensional plot of ω/ω_0, i.e., the nonlinear effects on the frequency, as a function of a/λ_D and ka. The frequency difference $\Delta\bar{\omega}(=\bar{\omega}-\bar{\omega}_0)$ for small wave numbers is illustrated in Figure 3. As we see in these figures, the nonlinear correction effects on the dispersion relation increase with decreasing a/λ_D due to the characteristic behavior of the exponential integrals $\mathrm{Ei}(-3a/\lambda_\mathrm{D})$ and $\mathrm{Ei}(-a/\lambda_\mathrm{D})$. It should be noted that the nonlinear correction effects enhance the wave frequency because the nonlinear correction terms in the modified interaction potential $U_{ij}(\mathbf{r}_i,\mathbf{r}_j)$ in (1) enhance the force acting on the jth dust grain. Then, it can be expected that the group velocity of transverse dust lattice mode waves would be increased due to the nonlinear correction effects. It is also found that the group velocity of the wave decreases with increasing a/λ_D. Hence, the nonlinear effect plays an important role in the investigation of the physical properties of dusty plasmas. These results provide useful information on the nonlinear effects on the transverse dust lattice waves in strongly coupled dusty plasmas.

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